Floor spectra for analysis of acceleration-sensitive equipment in buildings

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ABSTRACT:
In design codes and practice, not enough attention has been paid to the seismic resistance of acceleration-sensitive equipment installed in buildings. For the seismic design and assessment of such equipment floor acceleration spectra, which are based on an uncoupled analysis of the structure and the equipment, are commonly used. An “accurate” determination of floor spectra requires a complex and quite demanding dynamic response-history analysis. Recently a code-oriented method for the direct generation of floor acceleration spectra from design spectrum was developed, taking into account the structure’s dynamic properties. The method can be used for both elastic and inelastic multi-degree-of-freedom structures and equipment modelled as an elastic or inelastic single-degree-of-freedom oscillator. In this paper, the method is summarized and a test example of application is presented.

Keywords: floor acceleration spectrum, non-structural element, equipment, direct method, N2 method

1. INTRODUCTION
Typically, structural components of a commercial building account for approximately 15-25% of the original construction cost, while the non-structural components (mechanical, electrical, plumbing and architectural), account for the remaining 75-85% of the cost. Contents belonging to the building occupants, such as movable partitions, furniture, and office or medical equipment represent a significant additional value at risk. Consequently, the largest capital investment in most commercial buildings is in the non-structural systems and contents (FEMA E-74, 2012), which, in this paper, will be referred to as equipment. The importance of non-structural components is illustrated in Fig. 1 for three common types of commercial constructions. Nevertheless, in general, not enough attention has been paid to the seismic behaviour of equipment, with the exception of the nuclear industry, where the main concern is the seismic safety of the equipment.

![Figure 1. Typical investments in building construction (after FEMA E-74, 2012).](image)

For the seismic design and assessment of acceleration-sensitive equipment, floor acceleration spectra
are commonly used. These spectra can be applied when the mass of the equipment is significantly smaller than the mass of the primary structure, e.g. by a factor of more than one hundred, which is usually the case. Floor spectra depend both on the characteristics of the ground motion and those of the structure, and can be "accurately" determined by performing a response-history analysis of the structure. Since this approach is time-consuming, it is only used exceptionally, e.g. in the nuclear industry. In everyday design practice, an approximate approach is commonly used, where the floor spectra are determined directly from the design ground motion characteristics. Several methods of different complexity and with different limitations exist in this approach. They can be called "direct" methods. Building codes and standards (e.g. Eurocode 8, 2004, and ASCE 7-10, 2010) provide oversimplified formulae for the determination of floor spectra. There is an urgent need to improve the code procedures for the determination of floor spectra. Recently, a method for the direct estimation of floor acceleration spectra was developed by the authors (Vukobratović and Fajfar, 2015, 2016a, 2016b). The method can be used for both elastic and inelastic multi-degree-of-freedom structures and equipment modelled as an elastic or inelastic single-degree-of-freedom oscillator. Taking into account the inelastic behaviour of the building and/or of the equipment can greatly improve the economic aspects of equipment design. In the case of inelastic primary structures, the method is coupled with the pushover-based analysis method. The method is intended for practical applications, e.g. for implementation in guidelines and codes. In the paper, the method is summarized, and all the formulae needed for the calculation of floor acceleration spectra are provided. A description of all steps of the analysis, together with all the relevant numerical data, is presented in a test example. The paper represents a shortened and slightly modified version of a recent journal paper by the authors (Vukobratović and Fajfar, 2016b). More detailed results, including comparisons with the floor spectra obtained by response-history analysis, are provided in Vukobratović and Fajfar (2015, 2016a).

2. SUMMARY OF THE METHOD

In this section, the procedure for the determination of floor acceleration spectra is summarized, together with all the formulae needed for computations. The formulae apply to elastic equipment modelled as single-degree-of-freedom (SDOF) systems. The inelastic behaviour of equipment can be taken into account approximately by increasing its damping, as discussed in Section 2.2.3.

2.1 Initial calculations

First, an elastic modal response spectrum analysis of the building structure (referred to as the primary structure) has to be performed. If inelastic structural behaviour is taken into account, then a pushover-based analysis of the primary structure, e.g. by means of the N2 method implemented in Eurocode 8 (2004), is also needed (see Fajfar, 1999, 2000). If, for the seismic analysis of the primary structure, the equivalent lateral force method is used, in which the higher mode effects are neglected, then the proposed direct method can be used by considering the fundamental mode only.

2.2 Floor acceleration spectrum

The floor acceleration spectrum represents the maximum acceleration of the secondary system \( A_s \) as a function of the period of the secondary system \( T_s \). For each mode of the primary structure that is taken into account, the floor acceleration spectra are determined separately and then combined together in order to obtain the resulting floor response spectrum.

2.2.1 Floor acceleration spectra for individual modes

For mode (i) and floor (j), the value of the floor acceleration spectrum is determined as

\[
A_{ij} = \frac{\Gamma_i \phi_0}{\left(\frac{T_i}{T_p}\right)^2 - 1} \sqrt{\left(\frac{S_{eq}}{R_p}\right)^2 + \left(\frac{T_i}{T_p}\right)^2 S_m} \tag{1a}
\]

\[
\left|A_{ij}\right| \leq AMP \times |PFA| \tag{1b}
\]
\[
PFA_{ij} = \Gamma_i \phi_j^i S_{ep,i} / R_{\mu}
\]

\[
AMP_i = \begin{cases} 
2.5 \sqrt{10/(5 + \xi_s)}, & T_{p,i} / T_c = 0 \\
\text{linear between } AMP_i \left(T_{p,i} / T_c = 0\right) \text{ and } AMP_i \left(T_{p,i} / T_c = 0.2\right), & 0 \leq T_{p,i} / T_c \leq 0.2 \\
10 / \sqrt{\xi_s}, & T_{p,i} / T_c \geq 0.2
\end{cases}
\]

Eq. 1a corresponds to the off-resonance region. The formula is based on the principles of structural dynamics. The plateau of the floor acceleration spectrum in the resonance region, where the period of the equipment is equal or similar to the period of the primary structure, is determined as a product of the peak floor acceleration \(PFA_{ij}\) and an empirical amplification factor for the considered mode \(AMP_i\) (Eq. 1b). Since \(A_{s,ij}\) and \(PFA_{ij}\) can be either positive or negative, absolute values are used in Eq. 1b. The equation for the peak floor acceleration \(PFA_{ij}\) (Eq. 2) represents a special case of Eq. 1a, for \(T_s = 0\). Values of \(AMP_i\) can be determined from Eq. 3. The third line in Eq. 3 was proposed by Sullivan et al. (2013) and Calvi and Sullivan (2014). The equation is empirical. It yields values close to the more elaborated original formula proposed by the authors (Vukobratović and Fajfar, 2016a). The first line in Eq. 3 is based on the fact that in the case of an infinitely stiff structure \((T_e = 0)\) the equipment responds exactly in the same way as if it was situated on the ground. The formula represents the amplification at the plateau of the acceleration spectrum according to Eurocode 8 (2004). The second line represents a linear relation in the short period range, with the limit period which was determined empirically.

The indices "p" and "s" correspond to the primary structure and the secondary element, i.e. equipment, respectively. \(S_e\) is a value in the elastic acceleration spectrum which represents the seismic demand. \(S_{ep,i} = S_e(T_{p,i} \xi_{ep,i})\) applies to the \(i\)th mode of the primary structure, whereas \(S_{es} = S_e(T_{c} \xi_s)\) applies to the equipment. The natural periods of the \(i\)th mode of the structure and equipment are denoted as \(T_{p,i}\) and \(T_s\), respectively, whereas \(\xi_{ep,i}\) and \(\xi_s\) denote the damping values of the structure (for the \(i\)th mode) and of the equipment, respectively. In Eq. 3, \(T_c\) is the characteristic period of the ground motion (which is equal to \(T_c\) in Eurocode 8 2004), and \(\xi_s\) is expressed in \% of critical damping. \(R_{\mu}\) is the reduction factor due to inelastic behaviour of the primary structure (see Section 2.2.2). In the case of an elastic structure, \(R_{\mu} = 1.0\). \(\Gamma_i\) represents the modal participation factor of the \(i\)th mode, and \(\phi_j^i\) represents the \(i\)th mode shape coefficient at the \(j\)th floor. In the case of a simple planar structural model with concentrated masses, \(\Gamma_i\) is defined by Eq. 4, where \(m_j\) is the mass at the \(j\)th floor. Since in Eqs. 1 and 2 \(\Gamma_i\) is multiplied by \(\phi_j\), no normalization of the mode shapes is needed. Nevertheless, it is suggested that the mode shapes are normalized to 1.0 at the control point (usually at roof level) for transparency.

\[
\Gamma_i = \sum \frac{\phi_j^i m_j}{\phi_j^i m_j}
\]

2.2.2 Inelastic primary structure

It is assumed that inelastic behaviour of the structure is related only to the fundamental mode, where the mode shape \(\{\phi_i\}\) is represented by the inelastic deformation shape, whereas all higher modes are treated as elastic. Inelastic structural behaviour is taken into account through a ductility dependent reduction factor \(R_{\mu}\), which represents the ratio between the value of the elastic and inelastic spectrum for the relevant ductility demand \(\mu\). The reduction factor according to Eq. 5 proposed by Vidic et al. (1994), which has also been implemented in Eurocode 8 (2004), can be used.

\[
R_{\mu} = \begin{cases} 
\frac{T_p}{T_c} (\mu - 1) + 1, & T_p < T_c \\
\mu, & T_p \geq T_c
\end{cases}
\]

\(T_p\) represents the effective natural period, i.e. the period of the idealized equivalent SDOF system. Eq. 5 applies to the fundamental mode, whereas for all higher modes \((i > 1)\), \(R_{\mu}\) is taken as 1. In the case of a positive post-yield stiffness, it is necessary to divide the value of \(R_{\mu}\) defined by Eq. 5 by \((1 + \alpha(\mu - \ldots)\).
A pushover-based procedure, e.g. the N2 method, is needed in order to determine some parameters related to the inelastic structure. These are the inelastic deformation shape, the effective natural period $T_p$, and the ductility demand $\mu$. $T_p$ has to be used in Eqs. 1-3 instead of $T_{p,1}$. Moreover, an inelastic deformation shape, normalized to 1.0 at the control point (usually at roof level) has to be used in Eqs. 1 and 2 instead of the fundamental mode shape $\{\phi_1\}$. $T_{1}$ should also be determined from the inelastic deformation shape.

The determination of the reduction factor $R_y$ can be avoided if the quotient $S_{y0}/R_y$ for the first mode ($i = 1$), is replaced by the acceleration of the inelastic primary structure determined as $S_{y0} = F_y/m^*$, where $F_y$ and $m^*$ are the yield strength and the mass of the equivalent SDOF system, respectively, determined according to the rules of the N2 method (Dolšek, 2017). In the case of a positive post-yield stiffness, it is necessary to multiply the value of $S_{y0}$ by $(1 + \alpha(\mu - 1))$.

2.2.3 Inelastic equipment

Inelastic behaviour of ductile equipment reduces the floor acceleration spectra. This beneficial effect can be approximately taken into account by increasing the damping of the equipment. A very recent study conducted by the authors (selected results are provided in Vukobratović and Fajfar, 2016b) indicated that floor acceleration spectra for elastic equipment with 10% and 20% damping approximately correspond to the spectrum for inelastic equipment in the case of a ductility demand $\mu_s$ equal to 1.5 and 2.0, respectively, and the actual damping of the equipment equal to 1%. It was also found that, with increasing inelastic behaviour of the equipment, the influence of its damping rapidly decreases. Based on these observations, and considering that, in practice, it is hardly possible to make a reliable estimation of the damping and ductility of equipment, as a preliminary option, the following suggestion is made: for inelastic equipment, the procedure developed for elastic equipment (Eqs. 1-3) should be used by taking into account $\zeta_s = 10\%$, independently of the actual damping of equipment. This approach generally provides fair estimates of the floor acceleration spectra if the actual ductility $\mu_s$ is 1.5 and the actual equipment damping is about 1%. The results are generally conservative in the case of higher ductility and/or higher damping.

2.2.4 Combination of the floor spectra for individual modes

The floor acceleration spectra calculated for individual modes should be combined together in order to determine the resulting floor spectra. In the range of the periods of equipment from $T_i = 0$ up to and including the end of the plateau of the resonance region of the fundamental mode ($T_i = T_{p,1}$), the direct floor spectra obtained for individual modes should be combined by using the standard SRSS or CQC modal combination rules. In the post-resonance region of the fundamental mode (the rest of the period range), the algebraic sum (ALGSUM) should be used, in which the relevant signs of individual modes are taken into account. The upper limit of the resulting floor spectrum calculated from the ALGSUM is represented by the plateau obtained for the resonance region of the fundamental mode by using the SRSS or CQC rules.

3. EXAMPLE OF THE APPLICATION OF THE PROPOSED DIRECT METHOD

A three-storey reinforced concrete frame structure known as the "SPEAR building" was used as an illustrative example. Floor spectra are determined for the variant of the building re-designed in compliance with Eurocode 8 (2004) as a high ductility class structure (see Rozman and Fajfar, 2009, variant EC8 H). Characteristics of the building and of the structural modeling are provided in Rozman and Fajfar (2009). The floor spectra are determined for elastic equipment with 1% damping. As a variant, inelastic response of the equipment (with a ductility factor $\mu_s$ equal to 1.5) is also considered. Ground motion is defined by the type 1 elastic spectrum for soil type B as defined by Eurocode 8 (2004). The peak ground acceleration $PGA$ is equal to 0.35g, where the soil factor is already included.

In the following text, all the steps of the analysis considering ground motion action in the negative x-direction are presented. For simplicity, the floor spectra are determined only at the mass centres (denoted as CM in Fig. 2), where torsional influence is negligible.
3.1 Free vibration analysis of the primary structure

The spatial model of the primary structure can vibrate in 9 modes. Three of them have predominant components in the x-direction. The natural periods $T_{p,i}$ (the index i denotes the vibration mode), the mode shapes, and the $\Gamma_i$ values (Eq. 4) for these three modes are presented in Table 1. The effective period, deformation shape and transformation factor (in this case equal to the modal participation factor) obtained by the N2 method for the inelastic fundamental mode (see Section 3.2) are also presented.

![Diagram of SPEAR building](image)

Figure 2. Plan and cross-section of the SPEAR building (all dimensions are in meters).

Table 1. Natural periods, mode shapes and modal participation factors of the test structure (x-direction).

<table>
<thead>
<tr>
<th>Mode (i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{p,i}$ [s]</td>
<td>0.56</td>
<td>0.19</td>
<td>0.12</td>
<td>0.61</td>
</tr>
<tr>
<td>$\phi_{i1}$</td>
<td>0.33</td>
<td>-1.16</td>
<td>2.83</td>
<td>0.36</td>
</tr>
<tr>
<td>$\phi_{i2}$</td>
<td>0.69</td>
<td>-0.86</td>
<td>-2.55</td>
<td>0.70</td>
</tr>
<tr>
<td>$\phi_{i3}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>1.28</td>
<td>-0.34</td>
<td>0.08</td>
<td>1.28</td>
</tr>
</tbody>
</table>

3.2 Pushover (N2) analysis of the primary structure

Pushover analysis is needed if the beneficial effect of the energy dissipation capacity due to ductile inelastic seismic response of the primary structure is to be taken into account. The N2 method, as implemented in Eurocode 8 (2004), is used. The lateral load pattern corresponds to the first mode shape.

The calculated and idealized pushover curves are presented in Fig. 3a. The inelastic deformation shape is shown in Table 1. The mass of the equivalent SDOF system is equal to $m^* = 192$ t. The transformation factor $\Gamma_i$ which in the N2 method is equal to the modal participation factor $\Gamma_1$ obtained from the elastic modal analysis (Eq. 4) (provided that the lateral loads are based on the fundamental mode shape, and that this mode shape is properly normalized – the value at the control point, usually at roof level, is equal to 1.0), is equal to 1.28. Note that in the N2 method, the assumed displacement shape (obtained from the elastic modal analysis) is used for the determination of the transformation factor $\Gamma_i$, whereas, in the case of an inelastic structure, $\Gamma_i$ according to Eq. 4 is determined from the inelastic deformation shape, which represents the calculated displacement shape in the first iteration of the pushover procedure. The
differences in $\Gamma$ values obtained from the assumed and calculated displacement shapes are usually small (in the investigated case, both $\Gamma$ values are equal – see Table 1). The yield force and the yield displacement of the idealized equivalent SDOF system are equal to $F^*_y = 546$ kN and $D^*_y = 2.7$ cm, respectively. The capacity diagram, i.e. the pushover curve for the equivalent SDOF system in acceleration-displacement (AD) format, is shown, together with the demand spectrum, in Fig. 3b. As presented in Fig. 3b, the seismic demand for the equivalent SDOF is equal to $D^*_t = 6.6$ cm, whereas the demand in terms of the roof displacement is equal to $D_t = \Gamma D^*_y = 8.4$ cm. The ductility demand is equal to $\mu = 2.4$. Since the period of the idealized system, i.e. the effective period, is equal to $T^*_p,1 = T^*_p = 0.61$ s, which is larger than the characteristic period of the ground motion $T_C = 0.5$ s, the reduction factor is equal to the ductility demand: $R_\mu = \mu = 2.4$ (Eq. 5). In the case of inelastic behaviour, $T^*_p,1$ will be used in Eqs. 1-3 instead of $T_{p,1}$, as mentioned in Section 2.2.2.

Figure 3. Results of the analysis with the N2 method: (a) calculated and idealized pushover curves for the MDOF system, and (b) capacity diagram, demand spectrum and displacement demand.

### 3.3 Peak floor accelerations

Peak floor accelerations $PFA_{ij}$ for the three elastic modes of vibration and for the inelastic fundamental mode are determined by Eq. 2. The resulting $PFA$ values for the elastic (using three elastic modes) and inelastic (using the inelastic fundamental mode and the elastic second and third modes) primary structure are obtained by the SRSS combination rule. The results are presented in Table 2. Note that the $PFA$ values represent the values in the floor acceleration spectra which correspond to infinitely stiff equipment ($T_s = 0$). A considerable reduction in the inelastic $PFA$ values can be observed.

<table>
<thead>
<tr>
<th>Mode (i)</th>
<th>Elastic</th>
<th>Inelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$S_{ep,i}$ [g]</td>
<td>0.78</td>
<td>0.87</td>
</tr>
<tr>
<td>$R_\mu$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$PFA_{i1}$ [g]</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>$PFA_{i2}$ [g]</td>
<td>0.69</td>
<td>0.25</td>
</tr>
<tr>
<td>$PFA_{i3}$ [g]</td>
<td>1.0</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

Note that $S_{ep} = F^*_y/m^* = 0.29g$ (see Fig. 3b) is practically equal (a small difference exists because of the rounding error) to $S_{ep}/R_\mu = 0.71g/2.4 = 0.30g$ (see Table 2 and Section 2.2.2).
3.4 Determination of floor spectra for individual modes

Floor acceleration spectra values for the $i^{th}$ mode at the $j^{th}$ floor ($A_{s,ij}$) are obtained from Eq. 1. The amplification factors $AMP_i$ (Eq. 3) for 1% damping of the elastic equipment is equal to 10 for all three elastic modes, as well as for the fundamental mode of the inelastic primary structure (in all cases the ratio $T_{p,i}/T_C$ exceeds 0.2). The spectra for the 1st floor are plotted in Fig. 4.

![Figure 4. Floor acceleration spectra at the 1st floor for three modes of the elastic structure and for the fundamental mode of the inelastic structure. 1% damping of elastic equipment is taken into account.](image)

3.5 Determination of the floor spectra

The floor acceleration spectra obtained for the individual modes in the previous step (Fig. 4) need to be combined in order to determine the resulting floor spectra, as described in Section 2.2.4. In the case of the elastic primary structure, the spectra for the three elastic modes are combined, whereas in the case of the inelastic primary structure, the spectra corresponding to the fundamental inelastic mode are combined with the elastic spectra for the second and third mode. In Fig. 5 the resulting floor acceleration spectra are presented for both the elastic and inelastic primary structure for the first and the third floor.

The floor spectra presented in Fig. 5 demonstrate that a broadening of the spectra in the resonance region, which is required by standards due to uncertainties in assessing natural periods, is already implicitly included in the method, especially in the case of the fundamental mode.

In the case that the equipment is allowed to deform in the inelastic range and dissipate energy, the same procedure for the determination of floor acceleration spectra is used, but the damping of the equipment is set to 10%, assuming that the ductility capacity of the equipment ($\mu_s$) is equal to at least 1.5. The resulting floor spectra presented in Fig. 5 demonstrate a substantial reduction in the accelerations compared to the floor spectra for elastic equipment, especially in resonance regions.

A comparison with the Eurocode 8 floor spectra is provided in Vukobratović and Fajfar (2016b).

4. CONCLUSIONS

A practice-oriented method for the direct generation of floor acceleration spectra in buildings has been developed (Vukobratović, 2015, and Vukobratović and Fajfar, 2015, 2016a, 2016b). It is based on the theory of structural dynamics, in combination with empirically determined values for amplification factors in the resonance region. Higher mode effects are taken into account. Inelasticity of the structure is considered only in the fundamental mode, whereas higher modes are treated as elastic. Inelastic behaviour of the equipment is taken into account by increased damping. The modal superposition approach is involved in the method’s formulation. Even though not theoretically correct in the case of inelastic structures, it is considered to be an acceptable approximation.

The validation of the original method demonstrated that the method provides very good results in the case of elastic primary structures, whereas, in the case of inelastic primary structures, the results are an appropriate approximation, which can be used at least for preliminary analyses and for checking the results of more detailed analyses. Being fairly simple and reasonably accurate, the proposed method
represents a useful alternative to the extremely rough procedures that are used for the estimation of floor spectra in different codes.

![Diagram showing floor acceleration spectra for different types of buildings and equipment.](image)

**Figure 5.** Floor acceleration spectra for the elastic and inelastic ($\mu = 2.4$) structure and elastic and inelastic ($\mu_s = 1.5$) equipment. The damping of elastic equipment is equal to 1%.

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